Introduction to R-INLA

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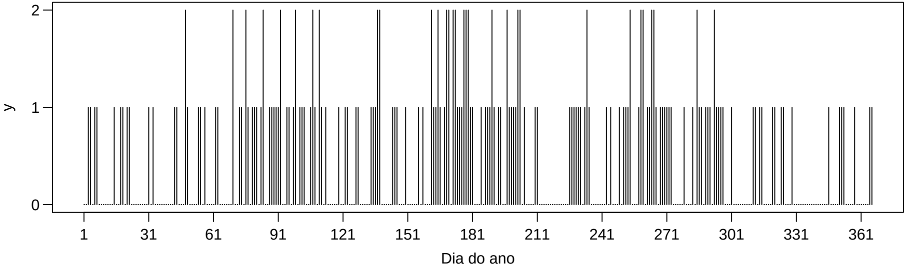
Example 1: Rain each day over two years by each day of the year



Problem: model the probability of rain each day of the year

Model

- y_i assume values 0, 1 or 2, for $i = 1, ..., n$
  - assuming conditional independence, thus

$$y_i | p_i \sim \text{Binomial}(2, p_i)$$

  - link function (logit)

$$p_i = 1/(1 + \exp(-x_i))$$

- **x**: smoothing over time
  - ***R**andon **W**alk* - RW of first order: `rw1`
  - Gaussian distribuiton for the sucessive differences (**R** esparse)

$$\Delta_i = x_i - x_{i-1} \sim N(0, \tau^{-1})$$

$$\log(\pi(\mathbf{x}|\tau)) \propto -\frac{\tau}{2}[(x_1 - x_n)^2 + \sum_{i=2}^{n}(x_i - x_{i-1})^2] = -\frac{\tau}{2}\mathbf{x}'\mathbf{R}\mathbf{x},$$

  - **x** is a *Gaussian Markov Random Field* - GMRF, Rue and Held (2005)
- τ: local precision parameter

Model, INLA

- Model

$$\begin{aligned} y_i|x_i &\sim \text{Binomial}(2, p_i) & \rightarrow \text{likelihood} \\ \mathbf{x}|\tau &\sim N(\mathbf{0}, (\tau\mathbf{R})^{-}) & \rightarrow \text{latent field, GMRF} \\ \tau &\sim p(\tau) & \rightarrow \text{prior distribution} \end{aligned}$$

- INLA: Acronym of ***I**ntegraged* ***N**ested* ***L**aplace* ***A**proximation*, Rue, Martino, and Chopin (2009)
  - First ***L**aplace* ***A**pproximation* - LA to compute

$$\pi(\tau|\mathbf{y})$$

  - Second (nested to the first) LA to compute

$$\pi(x_i|\mathbf{y})$$

- If the likelihood is Gaussian, the “approximation” is exact

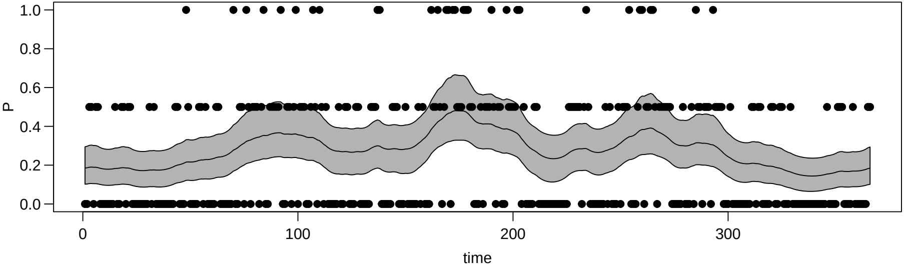
Example of a time series with rw1

```r
head(Tokyo, 5)
```

```
##   y n time
## 1 0 2    1
## 2 0 2    2
## 3 1 2    3
## 4 1 2    4
## 5 0 2    5
```

```r
model <- y ~ f(time, model='rw1', cyclic=TRUE)
result <- inla(model, family='binomial',
               data=Tokyo, Ntrials=n,
               control.compute=list(cpo=TRUE))
```

Result for the time series



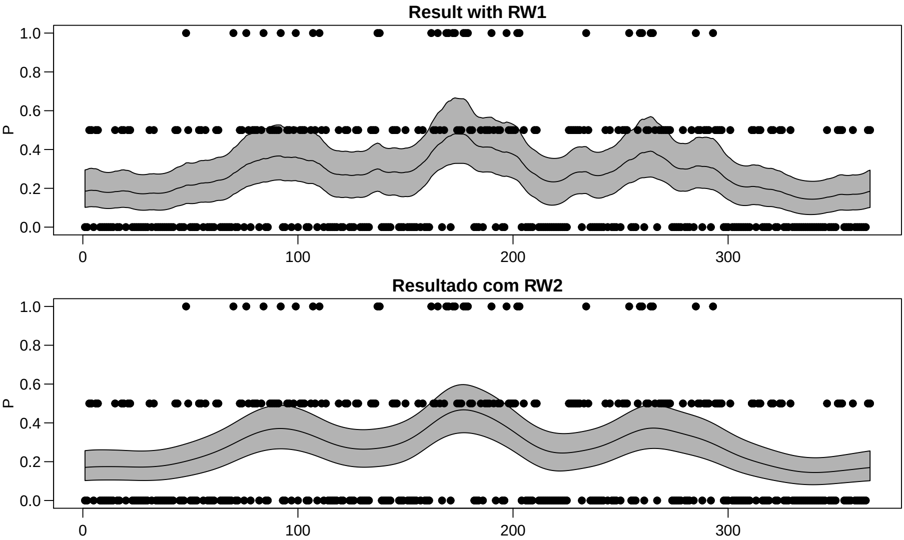
Smoothing more

Gaussian distribution for the second order differences (`rw2`)

$$\Delta_i^2 = x_i - 2x_{i-1} + x_{i-2} \sim N(0, \tau^{-1})$$

```
model2 <- y ~ f(time, model='rw2', cyclic=TRUE)
result2 <- inla(model2, family='binomial',
                data=Tokyo, Ntrials=n,
                control.compute=list(cpo=TRUE))
```

Both results for the time series



Goodness-of-fit measures

- ***C**onditional **P**redictive **O**rdinate* - CPO:

$$P(y_i^{\text{obs}}|\mathbf{y}_{-i})$$

$\mathbf{y}_{-i}$ is the $\mathbf{y}$ vector without the y_i element.

```
c(-sum(log(result$cpo$cpo)),
  -sum(log(result2$cpo$cpo)))
```

```
## [1] 312.6621 314.3368
```

- ***P**robability **I**ntegral **T**ransform* - PIT:

$$P(Y_i \leq y_i^{\text{obs}}|\mathbf{y}_{-i}).$$

- Useful to detect lack of fit or outliers

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Possibilities of models

- Generalized (mixed) models
- Generalized additive (mixed) models
- Survival models
- Dynamic models
- Stochastic volatitily models
- Smoothing spline
- Semiparametric regression
- Disease mapping
- Model based geostatistics*
- Log-Gaussian Cox processes
- Space-time models
- Semiparametric regression with spatial (space-time) varying coefficients
- +++

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$\rightarrow$ GLMM, GAM, GAMM, ... are different names for the same thing

Flexibility and responsability

- In the rainfall ocurrence example, suppose we add other terms (covariate, seazonal effect)
- Flexibility must come with responsability
- PC-prior ***P**enalized **C**omplexity* prior, Simpson et al. (2017)

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Implementation

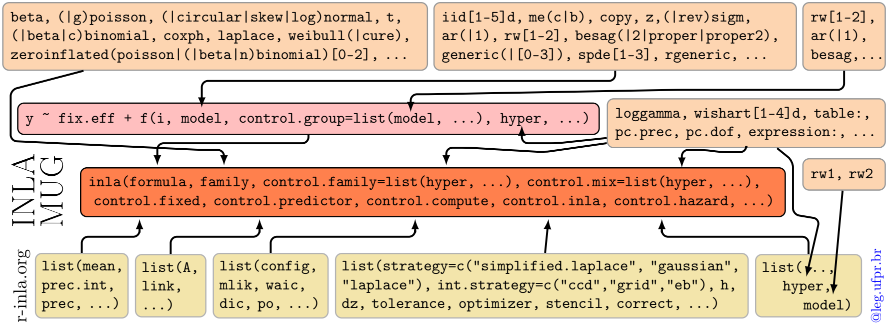
The code

- gmrflib, inlaprog, fmesher
  - ~6000 lines in Fortran
  - +100000 lines in C++
- external libraries: muparser, taucs (sparce matrices), PARDISO (experimental)
- ~60000 lines of code+help in R (R-INLA)

The work

- +15 years Håvard Rue (~80% of the code)
- +10 years Finn Lindgren (~20% of the code)
- source code in the bitbucket (open source), web page: r-inla.org

Main function in R-INLA



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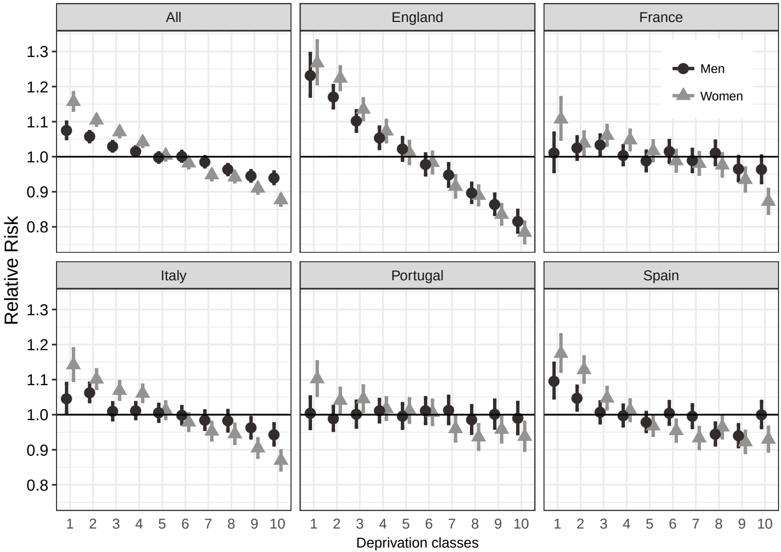
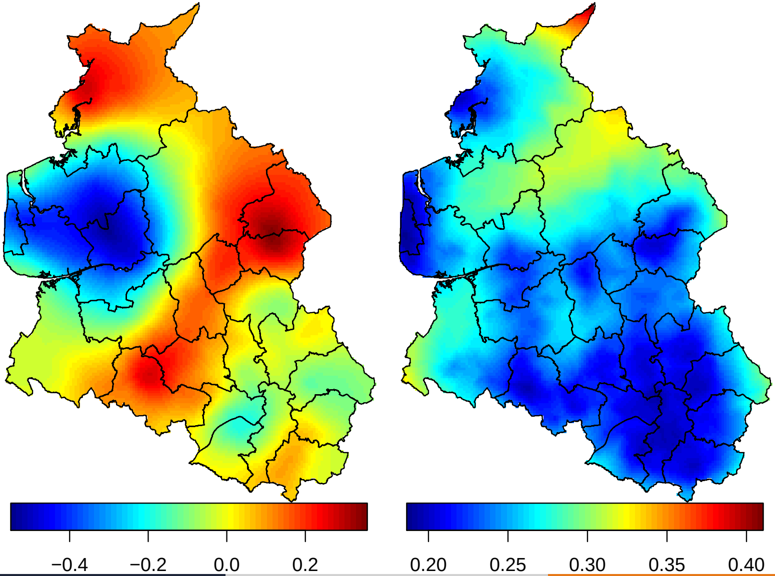
Some applications cited in Rue et al. (2017)

Recent examples of applications using the R-INLA package for statistical analysis include disease mapping (Schrödle & Held 2011a , b ; Ugarte et al. 2014 , 2016 ; Papoila et al. 2014 ; Goicoa et al. 2016 ; Riebler et al. 2016); age-period-cohort models (Riebler & Held 2016); a study of the evolution of the Ebola virus (Santermans et al. 2016); the relationships between access to housing, health, and well-being in cities (Kandt et al. 2016); the prevalence and correlates of intimate partner violence against men in Africa (Tsiko 2016); a search for evidence of gene expression heterosis (Niemi et al. 2015); analysis of traffic pollution and hospital admissions in London (Halonen et al. 2016); early transcriptome changes in maize primary root tissues in response to moderate water deficit conditions by RNA sequencing (Opitz et al. 2016); performance of inbred and hybrid genotypes in plant breeding and genetics (Lithio & Nettleton 2015); a study of Norwegian emergency wards (Goth et al. 2014); effects of measurement errors (Muff et al. 2015 , Muff & Keller 2015 , Kröger et al. 2016); network meta-analysis (Sauter & Held 2015); time-series analysis of genotyped human campylobacteriosis cases from the Manawatu region of New Zealand (Friedrich et al. 2016); modeling of parrotfish habitats (NC Roos et al. 2015); Bayesian outbreak detection (Salmon et al. 2015); long-term trends in the number of Monarch butterflies (Crewe & Mccracken 2015); long-term effects on hospital admission and mortality of road traffic noise (Halonen et al. 2015); spatio-temporal dynamics of brain tumors (Iulian et al. 2015); ovarian cancer mortality (García-Pérez et al. 2015); the effect of preferential sampling on phylodynamic inference (Karcher et al. 2016); analysis of the impact of climate change on abundance trends in central Europe (Bowler et al. 2015); investigation of drinking patterns in US counties from 2002 to 2012 (Dwyer-Lindgren et al. 2015); resistance and resilience of terrestrial birds in drying climates (Selwood et al. 2015); cluster analysis of population amyotrophic lateral sclerosis risk (Rooney et al. 2015); malaria infection in Africa (Noor et al. 2014); effects of fragmentation on infectious disease dynamics (Jousimo et al. 2014); soil-transmitted helminth infection in sub-Saharan Africa (Karagiannis-Voules et al. 2015); analysis of the effect of malaria control on *Plasmodium falciparum* in Africa between 2000 and 2015 (Bhatt et al. 2015); adaptive prior weighting in generalized regression (Held & Sauter 2016); analysis of hand, foot, and mouth disease surveillance data in China (Bauer et al. 2016); estimation of the biomass of anchovies in the coast of Perú (Quiroz et al. 2015); and many others.

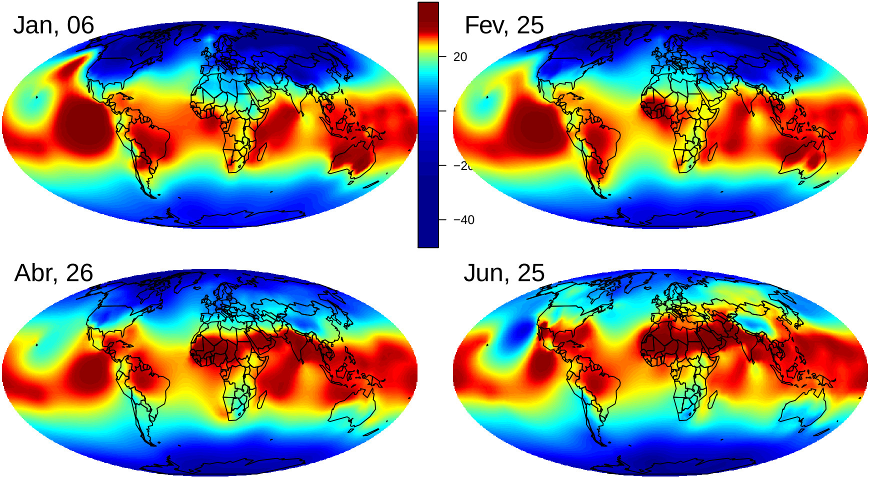
Deprivation effect, Ribeiro et al. (2018)



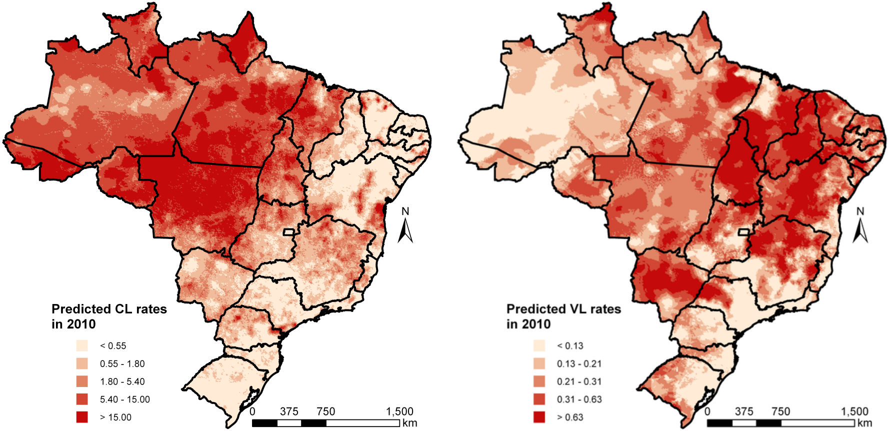
Survival: frailty map



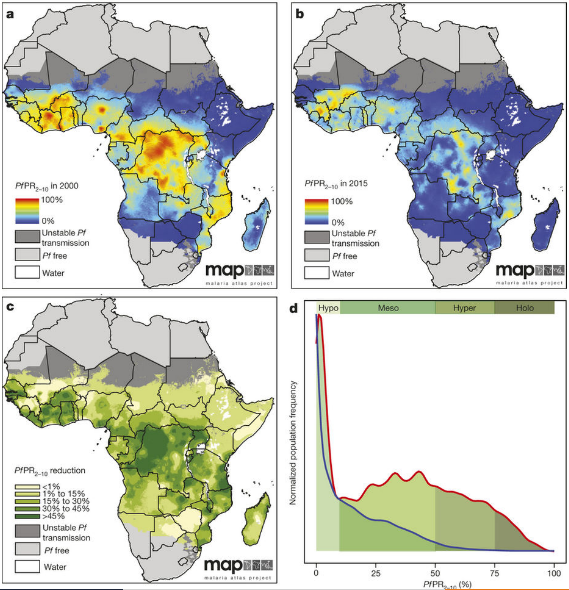
Non-separable space-time modeling in the globe



Leishmaniasis in Brazil, Karagiannis-Voules et al. (2013)



Malaria in Africa, Gething (2015)



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